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Logrolling and Strategic Preferences for High-Transaction-Cost Legislative Dynamics: A Quid Pro Quo Level-k Approach

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Abstract

This paper introduces a proposal to consolidate concepts from different branches of economics, such as logrolling and the theory of levels of k-reasoning, to argue how legislators' preferences might vary in a mutually beneficial exchange of votes. It explores the possible forms that utility functions could take under transaction cost contexts such as ideological distance and delay in the processing of a bill proposal. It highlights arguments suggesting that a higher k-level could lead legislators to tend toward voting in favor of a coalition. Additionally, it incorporates a logrolling logic model through the conditional probability that the same coalition forms over two consecutive periods. Simultaneously, the article provides a historical overview of logrolling studies and legislative dynamics, considering both psychosocial and behavioral aspects of social choice.

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Intercambio de votos y preferencias estratégicas en dinámicas legislativas con altos costos de transacción: Un enfoque de quid pro quo basado en niveles-k

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Resumen

Este artículo introduce una propuesta para consolidar conceptos de diferentes ramas de la economía, como el intercambio de votos (*logrolling*) y la teoría de niveles de razonamiento-k, con el objetivo de argumentar cómo pueden variar las preferencias de los legisladores en un intercambio mutuo y beneficioso de votos. Se exploran las posibles formas que podrían adoptar las funciones de utilidad en contextos de costos de transacción, como la distancia ideológica y los retrasos en la tramitación de una propuesta legislativa. Se destacan los argumentos que sugieren que un nivel-k más alto podría llevar a los legisladores a inclinarse por votar a favor de una coalición. Además, se incorpora un modelo lógico de *logrolling* mediante la probabilidad condicional de que la misma coalición se forme en dos periodos consecutivos. Simultáneamente, el artículo ofrece una revisión histórica de los estudios sobre *logrolling* y dinámicas legislativas, considerando aspectos psicosociales y de elección social conductual.

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1. INTRODUCTION

When analyzing economic transactions within legislative procedures, votes can be interpreted as the currency of exchange among legislators. While legislative processes vary across countries and congressional systems, it is possible to identify analytical mechanisms that, through economic concepts, allow for a deeper understanding of voting behavior, particularly in contexts where mutual benefit and cooperation are central goals.

The main objective of this paper is to develop a theoretical framework that explains legislators' voting behavior as a strategic exchange process. This framework is grounded in the economic concept of logrolling and the principles of cognitive hierarchies from level-k reasoning. The research question addressed is: how do strategic reasoning and transaction costs influence the formation of legislative coalitions through vote trading mechanisms? To explore this question, the study employs an interdisciplinary approach that integrates public choice theory, behavioral economics, and strategic game theory. Using level-k reasoning, the paper investigates how legislators' expectations of others' actions shape their decision-making processes and influence coalition-building within the context of vote trading.

At the core of the analysis is the idea that legislative logrolling can reflect a level-k adaptation in the formation of coalitions or voting strategies, particularly under majority rule systems. The relevance of these dynamics varies depending on the institutional and electoral contexts, including the rules reinforced by a congress or embedded in a country's constitution. This study aims to identify key relationships between legislative votes, the incentives shaping them, and two specific transaction costs that legislators face: ideological reasoning, which refers to the ideological distance between legislators that affects their willingness to cooperate, and delay in legislative processing, which captures the impact of time on strategic decision-making and coalition formation.

The concept of logrolling, defined as the reciprocal exchange of votes between legislators, is used to examine how preferences are expressed and coordinated. In these exchanges, legislators rely on informal networks, subtle signaling, and tacit understandings to reach agreements that may lead from suboptimal to Pareto-optimal outcomes (Qu & Cheung, 2012). By incorporating level-k reasoning into the analysis, the paper models how legislators with varying cognitive levels and ideological preferences navigate transaction costs such as delays, ideological distance, or coalition difficulties. These costs are represented through adaptive utility functions, allowing for the exploration of how legislative cooperation emerges or fails. Ultimately, this model provides insights into the allocation of resources and political favors—key elements in understanding decision-making and strategic behavior in legislative environments.

2. LITERATURE REVIEW

The study of legislative decision-making spans various fields within social choice theory. Formal investigative approaches have aimed to focus on the contexts of Anglo-Saxon countries, evaluating congressional behavior and seeking to determine legislators' preferences regarding voting for or against proposed legislation. The objective of studying variations in legislative voting is to explore additional causes or impacts of factors external to the voting process.

For instance, Chappell Jr. (1981) found no evidence suggesting that congressional votes depend on private financial interests. In his work *Conflict of interest and congressional voting: A note*, he discusses how conflicts of interest among legislators significantly affect their voting behavior through a logit econometric model. However, factors such as ideology or the legislator's urban area could influence their decision-making. Similarly, Kau and Robin (1979) found similar results regarding individual interests and voter ideology using a linear probability regression model. Moreover, Laband (1988) highlights transaction costs in the competitive design of the legislature through ordinary least squares regression, finding that the duration of legislative sessions could be affected by legislative majorities and the level of party domination in the process.

This aspect is particularly relevant to the contribution of this manuscript, as transaction costs may alter legislators' decisions and preferences in contexts of uncertainty or delay in the passage of laws. Wood and Bohte (2004) also consider political transaction costs in administrative design, concluding that temporal delays in coalition formation could manipulate the formation of federal administrative agencies during 1879-1988 in the United States. This serves as a complementary study of the theory developed for legislative elections up to that point.

For societies under democratic regimes, understanding voting rules that conjecture legislative decisions is crucial to understanding the electoral outcomes of legislators and their impact on a bill. Ingham (2016) illustrates the importance of legislative majorities in achieving political objectives through law passage. Ensuring that legislators' preferences are expressed in each vote requires voting schemes capable of reflecting the honest expression of individual preferences. Gibbard (1973) argued in favor of validating such schemes as dominant strategies over alternatives, particularly in non-trivial situations.

The starting point for describing a mutually beneficial situation in legislative voting should focus on internalizing negative externalities that may render a logrolling situation mutually beneficial. Bernholz (2012) conceives these negative effects in cyclic social preference formations as problematic for achieving a Pareto optimal point unless legislators can engage in logrolling in each vote. However, Riker (1973) had previously analyzed the paradox of vote trading, highlighting the conditions and possibility of cyclic majorities.

Despite this, Oppenheimer (1979) argues that logrolling mechanisms always entail a mutually beneficial exchange of votes. Carrubba and Volden (2000) contribute to the literature by indicating an electoral need for districts that represent these legislators, as politicians would prefer universal coalitions *ex ante* to ensure minimal cooperation. They also find that additional contingent members on a coalition can increase costs and reduce benefits.

The "all-inclusive logroll" situation became pivotal in the study of legislative dynamics for political science and public-choice political economy. However, it wasn't until the Nobel laureate in economics, James J. Buchanan, along with his co-author from the Virginia school, Gordon Tullock, began describing logrolling as a pervasive political process applicable in specific contexts. Buchanan & Tullock (1965) analysis advanced the mathematical modeling of conditions and assumptions for conceiving voting models. McKelvey (1979) explored general conditions of intransitivity in votes within balanced institutional orders, implying that constitutional orders shape sustainable Pareto-optimal solutions. Tullock (1970) suggested that this stability also depends on the probability of forming majorities that include the largest number of participants.

Buchanan & Tollison (1984) posited that vote allocation after agreements represents the most desirable vote exchange or trade. Similarly, they theorized on efficiency properties to find an optimal outcome resolving indeterminacies under a majority voting rule. Each voter's decision indicates the direction of their preferences, but not the intensity; however, vote trading could resolve this final uncertainty. Bentley (1967) had previously analyzed logrolling dynamics, noting that if legislature members couldn't or wouldn't exchange votes, the activities related to that axis of power would cease to function. These arguments were part of his treatise *The Process of Government* almost a century ago.

On another front, Casella and Palfrey (2019) sought to study the dynamics of vote exchange for multiple law proposals where two laws are voted on as a group for a finite number of proposals with separable legislator preferences. Most recent studies within the fields of experimental and behavioral economics incorporate the concept-solution of the 'level-k reasoning model' to evaluate the sincerest responses to a voting process. Bassi (2015) demonstrated that these responses did not always align with the best-response strategies, as it depends on the type of game conducted. Her research showed that plurality rule-voting encourages sophisticated but not insincere voting, while Borda-count voting leads to the highest level of insincere voting. A laboratory experiment that followed a similar line of inquiry is Casella & Palfrey (2020). They introduced a vote-trading model where voters were paired and exchanged votes whenever mutually beneficial. The dynamics ultimately converged to stable vote allocations and demonstrated that the collected data could hold predictive power for generating projections.

3. METHODS AND MATERIALS

The analysis draws on the framework of level-k reasoning to explore how legislators might form expectations about others' actions and adjust their voting behavior under conditions of uncertainty and high transaction costs. Rather than constructing a fully specified formal model, the paper proposes a set of conjectures regarding the potential shapes of legislators' utility functions when facing ideological distance and delays in legislative processing. These conjectures aim to approximate how such costs may affect legislators' incentives to form coalitions or engage in vote trading.

As a simplified illustrative case, the study presents a stylized two-period game involving a set of three legislators within a legislative bargaining setting. This framework serves to introduce a probabilistic perspective on how a coalition formed in an initial period could be sustained or reciprocated in a subsequent period, thereby reflecting the logic of logrolling. The conditional probability of repeated cooperation is discussed in relation to each legislator's reasoning level and the contextual transaction costs present.

In this context, it is crucial to connect the concept of logrolling with economic theory and the development of level-k reasoning in the field of experimental economics. This opens the door to further discussions on how experimental methods can help validate and refine these theoretical insights, especially as they pertain to strategic decision-making in political environments.

3.1. Logrolling in Economic Theory

In the neoclassical conception of microeconomics, assumptions are studied, such as comparability, where any economic agent can express preference or indifference between combinations of goods. Additionally, these bundles of goods would be ordered and preferred under transitivity, referring to the consistency among the mentioned preferences, ensuring that curves forming the graphical representations of voters' preferences do not intersect. Differentiability ensures that their utility functions can be differentiated to an order assigned by the model. For the analysis of a voting scheme converging to a coordinating equilibrium in the legislative corpus, studying the logrolling phenomenon from microeconomic theory is necessary to observe the dynamics of vote exchanges, as previously defined in existing literature.

A logrolling situation occurs when there are two mutually exclusive legislative proposals and there exist legislator preferences in a transitive ordering, meaning one proposal is preferred over another. Let these be:

$$(X, Y) \wedge (Z, W)$$

And the preferences of legislators for each pair are separable, indicating that the way a legislator values a bill is not influenced by the existence of other proposals they would choose regardless of their interest. Following this formulation, legislators can only choose one proposal from each pair. They would choose X over Y and Z over W, but when both pairs are considered together, the voter prefers the set of issues Y and W over the set of issues X and Z. This means that:

$$X \succ Y$$

$$Z \succ W$$

$$Y\ e\ W \succ X\ e\ Z$$

This situation represents logrolling, where individual preferences in each pair of issues do not align when both pairs are considered together. Social preferences are also defined by the voting rule employed according to Bernholz (2012), and these characterizations may or may not hold, as there could be situations of indifference between proposals. In this case, the simple majority rule governs the voting. To provide a more concise idea about the implication of logrolling and the social gain or loss it carries, an example of a payoff matrix for the decision of three legislators (A, B, C) on two bills (X and Y) is illustrated.

Table 1. Voting Scheme

	Legislator A	Legislator B	Legislator C
Issue X	6	-2	-2
Issue Y	-2	-2	6

Table 1 reflects the payoffs legislators would obtain if they supported the proposed bills. These payoffs represent the benefit or utility for the voter when a project passes or is approved by a simple majority. In a preliminary analysis, neither of the two proposals would pass because if we observe option X, legislators B and C would have no interest in voting for that option. The same occurs with Y when analyzing the strategies of A and B. Given this voting behavior, both proposals would have two votes against and one in favor, resulting in zero total utility for this society of voters, as explained by Mauerberg, Strachman, and Reami (2013). However, if legislators A and C cooperate through logrolling, vote swapping would allow them to generate an alternative Pareto-optimal outcome. This would occur because A would agree to vote in favor of proposal Y, and voter C would do the same with X.

In this case, both bills would be accepted, and the total social benefit would be 4, signifying an improvement.

Summarizing this example with the economic definitions previously provided for the logrolling dynamics, legislators *A* and *C* would have a change in their strategic preferences to choose differently from what they would have chosen previously, as they would gain a benefit by doing so. It is necessary to note that this dynamic is not always so straightforward, as there might be transaction costs not internalized by legislators, such as pre-formation of coalitions, the impossibility for legislators to reach mutually beneficial agreements, and an imminent problem of a voting cycle.

Table 2. Logrolling

	Legislator A	Legislator B	Legislator C	Total
Issue X	6	-2	-2	2
Issue Y	-2	-2	6	2
SB	4	-4	4	4

Table 3. Logrolling for a negative sum-game

	Legislator A	Legislator B	Legislator C	Total
Issue X	6	-4	-4	-2
Issue Y	-4	-4	6	-2
SB	2	-8	2	4

For instance, if the payments to legislators when voting for *X* or *Y* were different, let's say instead of the negative -2, we had a -4 for each legislator, the social benefit would still be negative. This is shown in Table 3, indicating that rather than finding a better outcome, the situation would worsen. Hence, logrolling would align with cardinal preferences, prioritizing characteristics based on their intensity rather than their specific position. This was originally stipulated in a simple majority voting, as studied in the literature of spillover effects and theorized for these contexts by Gordon Tullock in his paper "Problems of Majority Voting."

An additional crucial consideration for studying these dynamics is the potential for the formation of infinite behavioral voting cycles, which would be a clear signal of the intransitivity of preferences. According to Bernholz (1973), the individual transitivity of legislator preferences does not guarantee aggregated transitivity. This had previously been examined for democratic voting systems by Arrow (1983). Many scholars built upon his work to theorize about the voting paradox in logrolling contexts, as inefficiencies in collective

decision-making processes arise when the aggregation of individual preferences does not reflect an option preferred by the majority of legislators. This might even result in a situation where social benefit consecutively remains negative despite some legislators accruing individual benefits (Riker & Brams 1973).

3.2. Level-k-Reasoning Approach

Part of the paper's objectives, in addition to tracing the historical evolution in the study of logrolling, is to provide an approach from different branches of economic study, such as public-choice political economy and behavioral economics. This approach situates logrolling within a high-transaction-cost context, where political decisions can be modeled as coordination problems. Therefore, a relatively modern conception is employed to reach equilibriums in social choice, known as the "level-k-reasoning" theory of cognitive hierarchy, wherein participants in these games consider their peers' strategies as more or less akin to their own. The level-k-reasoning framework offers a means to achieve agreements beneficial to their individual or collective interests.

The limited rationality of legislators in making strategic decisions is contingent upon the political dynamics within legislative contexts. Incumbents have incentives to find solutions to pass proposed legislation. This involves coordinating actions in advance to achieve beneficial agreements within a potential coalition. In legislative decision-making, akin to various other aspects of social choice theory, decision-makers do not possess infinite reasoning capacity to rationally choose their best option by anticipating the actions of others. Legislators have a limited level of understanding regarding their peers' actions, which can influence their final vote.

The level-k reasoning theory proposes distinct levels for players to think that they have a "strategic reasoning with one more step than the others" (Zhang, 2021, p.5). It is theorized that at level 0, participants in the voting process will make decisions solely based on their personal preferences without considering the actions of other legislators. These players might make decisions randomly or according to a predetermined pattern, disregarding the strategies of other voters. Conversely, level 1 players assume that level 0 players act randomly and try to predict their actions, shaping their decisions based on what they know so far. However, these players do not consider that the decisions of other players (i.e., players of different levels) are also conditioned on their own. Level 2 players assume that some players might be level 1 and seek to predict their moves, while level 3 players do the same with level 2 players. This continues to escalate levels. Players presume each lower level possesses some degree of rationality or relatively predictable behavior. This behavior is part of decision-making models where some agents view others as less sophisticated agents than themselves, as indicated by De Clippel et al.

(2016), and can even result in better equilibriums than the well-known Nash Equilibrium (NA).

Applied to legislative bargaining, the concept aims to incorporate the idea that higher-level decision-makers would attempt to predict the actions of lower-level legislators and adjust their strategies accordingly, generating variations in their preferences within contexts with limited information. This process would continue until legislators are completely certain of their peers' decisions, as further prediction becomes unnecessary.

In a logrolling context, it would be possible to model a scenario with multiple legislators, each with different levels of knowledge and preferences regarding the outcome of legislative proposals. Furthermore, additional factors influencing preference formation could be implemented due to the costs and benefits that legislators may obtain through the outcome of legislative approval. Indeed, Bassi & Williams (2014) propose the inclusion of financial incentives for complex decision situations through a laboratory experiment of fifteen sessions to observe students' focal decisions regarding the transitive property of monetary gain. They use a prior model of behavioral decisions based on Nagel (1995), who highlights theoretical solution concepts in a study on the importance of levels of behavioral reasoning. Zhang (2021) also adapts the general level-k model to study student decisions regarding school-choice and introduces a naive belief setting, where students at a sophisticated level of reasoning report their true preferences.

When players face transaction costs, such as in the proposed core, they can rely on these hierarchical levels of cognitive reasoning to find solutions and agreements. In the case of TC existing due to ideological distortions, legislators could be represented by levels reflecting their ideological affinity on a scale where higher levels indicate greater ideological affinity. A legislator with a lower level might be more willing to form coalitions with those who share similar ideologies, while a legislator with a higher level might be more willing to negotiate with those of lower ideological levels if sufficient benefits are obtained. In instances of difficulty in generating coalitions, a higher-level legislator might be willing to associate with legislators of different levels if the sum of their combined levels exceeds a certain threshold, reflecting coalition formation despite differences.

Similarly, in contexts where TC is attributed to waiting time in the processing of a bill, a higher-level legislator might be willing to reduce their demand on a specific issue if the resolution time is excessively prolonged, while a lower-level legislator might not be as inclined to quickly compromise on their positions. Such adaptations could be explored through mathematical modeling or the incorporation of specific parameters to provide a more assertive exploration in attempting to theorize these dynamics.

3.3. Modelling Some Conjectures

While introducing high transaction costs for the complexities and difficulties that legislators face in reaching agreements, the level-k-reasoning perspective theory could serve as an alternative for examining the understanding of logrolling. To illustrate a scenario wherein legislators exhibit ideological distances and experience changes in their preferences over the time delay for the processing or discussion of a proposal, an attempt is made to model conjectures for the utility functions of legislators, each possessing varying levels of strategic reasoning. These levels are defined on a scale from 0 to K, with K representing the highest level of reasoning. Legislators at level 0 would choose according to their personal preferences, those at level 1 would seek to predict the actions of those at level 0 and adjust their decisions accordingly, and so forth up to level K, as explained before.

3.4. Ideological Reasoning

Regarding the ideological distances among legislators, we can employ a utility function that combines ideological proximity and levels of rationality to determine each legislator's inclination towards fostering cooperation and voting similarly. Assuming a legislative framework with N legislators, each possessing a level of ideological affinity on a scale from 0 to 1, where 1 denotes complete ideological alignment.

$$N = (L_1, L_2, L_3, \dots, L_n)$$

$$I = [0,1]$$

For each legislator, their level of ideological affinity, denoted as I_i is considered. A legislator with a lower level of affinity I_i would be more inclined to enter into a voting agreement with legislators whose I_j affinities are closer to their own. It can be inferred that as voters are more ideologically aligned, their utility will increase. Legislators with higher levels of rationality might be willing to collaborate with other levels if they perceive sufficient benefits or have the capacity to predict the actions of lower levels, what we could label as ideological rationality. Then, describing the type of function that can be used to center the analysis:

$$e^{-\delta(I_i - I_j)}$$

The proximity or similarity between two values, in this case, the ideological closeness of legislators i and j, when raised to a negative power of the absolute value of their ideological affinity, might yield a result that exponentially decreases as the absolute value increases. The inclusion of a parameter

δ enable to adjust the exponential decay rate of the function. With a larger δ, the function decays more rapidly when there is greater ideological proximity, as opposed to a gradual decay with a smaller parameter.

Now, to incorporate the levels of reasoning of the legislators:

$$e^{-\rho(K_i,K_j)\cdot(I_i-I_j)} \quad (1)$$

$$\rho(K_i,K_j) = \frac{1}{K_i + K_j + r} \quad (2)$$

Where *r* is a constant that adjusts the relative influence of the reasoning level in the ideological affinity perception. For example, the perception of how the other legislator would respond the logroll is adjusted by the other's legislator *k* level. Additionally, as the levels of reasoning increase, the exponential decay rate decreases. This implies that legislators with higher levels of reasoning perceive ideological differences as less significant compared to those with lower levels. It is important to mention that this formulation does not conceive two legislators with the exact same ideology, since that could be difficult to find. This means that the difference between $I_i$and I_j cannot be 0.

Therefore:

$$U_i = I_i \cdot K_i \cdot \sum_{j\neq i} e^{-\rho(K_i,K_j)\cdot(I_i-I_j)} \quad (3)$$

$$U_i = I_i \cdot K_i \cdot \sum_{j\neq i} e^{-\frac{I_i-I_j}{K_i+K_j+r}}$$

The multiplication of the ideological affinity for legislator *i* with their level of rationality weighted by the exponential sum of ideological similarities between legislators and their respective levels of reasoning accounts for the utility derived from logrolling. The greater the ideological affinity and the level of reasoning of legislator *i*, the higher the utility for that legislator. Regarding the summation used, it considers all other legislators different from *i*, considering their ideological affinity, their level of reasoning, and their perception of conceding on an agreement.

3.5. Delay Time Under K-Reasoning:

In this scenario, legislators with higher level-k might adjust their demands or preferences on a legislative proposal if they perceive the resolution time as excessively prolonged. They could lean towards the possibility of

compromising on certain aspects and voting favorably to expedite the process. Conversely, those with lower-level k would be less likely to alter their stance as the process extends. Assuming, as in the previous context, a set of *N* legislators with varying levels of reasoning, an adaptive utility function is proposed to reflect how time affects preferences and the willingness to form alliances when voting. The adaptation concerning waiting time would involve a gradual adjustment of legislators' preferences and could be conceptualized through a logistic sigmoid function to explain how waiting time (T) and the level of reasoning (K_i) would influence a legislator's willingness to compromise in negotiation. In this regard, recognizing that the gained utility would measure the willingness to concede for a joint vote among legislators, it can be inferred that such willingness to agree might increase or decrease depending on the waiting time and the adaptability levels (i.e., reasoning) of legislators. The characterizations of these functions are that they have a domain of all real numbers, with return (response) value commonly monotonically increasing but could be decreasing as well. So, defining the utility function by:

$$U_i = \frac{1}{1 + e^{-a \cdot (K_i - b) \cdot T}} \tag{4}$$

Where

$T \in [0,1]$

Being a and b constants embedded in the function, a determines the slope of the logistic curve, signifying how rapidly the willingness to compromise changes over time and the level of rationality. On the other hand, constant b represents the inflection point where the function changes direction and reflects the level K of reasoning at which the waiting time begins to have a greater effect on the willingness to compromise.

It can be observed that as the waiting time T increases, the function changes its value. When T is small, more agreements can be reached, achieving stability over a broader range. However, as T increases and the time cost becomes more present, legislators with higher levels of reasoning might opt for a coalition to facilitate logrolling. Legislators with lower levels might be less convinced of this, although commitment to a future vote may also be crucial. The parameters expressed in the logistic function can be utilized to adjust the shape of the logistic curve (exponential) up to a point where the time and impact for the willingness to form agreements are determined.

It is noteworthy that legislators may have limited information about the preferences and actions of others, which could influence their ability to predict behaviors and form coalitions.

3.6. Coordination in the Political Process

After outlining potential forms that legislators' utility functions may take within a coalition-building process, an additional critical step is necessary regarding the situation of logrolling. It is sought, in addition to incorporating levels of reasoning based on voters' cognitive differences, to secure coalitions across two consecutive periods. Thus, if one legislator agrees to vote in alignment with their counterpart, for logrolling to occur, the other legislator must reciprocate in the second period.

In this scenario, legislators anticipate reciprocal voting in the second period within a strategic negotiation game, considering the incentives that legislators, who were favored in the first period, possess to continue and respond similarly in the second period. Without this, coalitions might form under certain contexts, but would not culminate in logrolling. One could argue that as long as logrolling remains feasible, levels of reasoning will influence its formation and structure, including potential costs generated from a negative sum-game, as explained in the section of this paper that describes economic logrolling.

According to Stratman (1992), the costs of an already approved bill must be borne collectively by all participants in the voting process, irrespective of their individual decisions. Moreover, these costs are likely to be felt to a lesser extent among the "final" voters. In the subsequent description presented, this can be understood as a reassessment of the incentives that legislators must "respond" to logrolling. A certain degree of stability in logrolling, as Coleman (1966) and Tullock & Brennan (1981) suggest, can be achieved if there is credibility among the agents involved. Enelow (1986) contributes to the debate by proposing a model where each legislator forecasts the future using a random variable with a linear mean across present proposal alternatives. In a much simplistic view to illustrate a logrolling scenario, consider a set of three legislators across two time periods to reinforce the theoretical conjectures proposed in this study, a simplified decision-making model is developed under conditions of bounded rationality. This model aims to illustrate how legislative agreements may (or may not) be reached depending on the cognitive reasoning levels of the actors involved and the presence of transaction costs.

After describing potential forms of legislators' utility functions during coalition-building, the focus shifts to how cognitive limitations—represented via level-k reasoning—affect the stability of coalitions across two consecutive legislative periods. The scenario assumes that logrolling requires reciprocity: a legislator agreeing to support another's proposal in period one expects a reciprocal vote in period two. Without this reciprocal behavior, strategic coalitions may form temporarily but fail to yield lasting logrolling outcomes.

Inspired by Austen-Smith & Banks (1988), the model accounts for legislators' beliefs and anticipations in a strategic environment. Legislators must decide whether to honor previous commitments based on expected utility

gains, ideological proximity, and the cost of delays in policy outcomes. This introduces the bounded rationality component: legislators do not calculate optimal strategies with complete information, but reason based on simplified beliefs about others' actions.

To formalize this, consider a stylized game involving three legislators over two periods:

$$N = \{L_1, L_2, L_3\}$$

$$P_t = \{t_1, t_2\}$$

The coalition C_{t1} is a subset of the legislator set *N*, consisting of two members. This implies that in the first period, a coalition comprised of two out of three legislators was established. Considering the possibility of the same coalition forming again in the second period (logrolling), this introduces levels of reasoning using the level-k-thinking framework, along with the conditional probability that the identical coalition will reassemble in the second period:

$$C_{t1} \subseteq N$$

$$|C_{t1}| = 2$$

Over a C_{t1} formation it is specified that $k = \{1,2\}$ and $K = 2$. In the equation above, *k* refers to the level of the legislator in question and *K* denotes the highest level of reasoning. The conditional probability would be described as:

$$P(C_{t2} = C_{t1} | C_{t1})$$

$$P_C^{(k)} = \frac{k}{K}$$

Where $P_C^{(k)}$ represents the probability based on the k-level of reasoning for the coalition to reoccur.

$$P(C_{t2} = C_{t1}|C_{t1}) = P_C^{(k)} \cdot P_{IgD}$$

$$P(C_{t2} = C_{t1}|C_{t1}) = P_C^{(k)} \cdot P_{td}$$

$$P_{IgD} = 1 - (I_{L_1} - I_{L_2})$$

$$P_{td} = e^{-\mu(T)}$$

P_{IgD} and P_{td} are the probabilities under TC contexts for ideological distance and time delay, respectively. Ideological distance is normalized within a range from 0 to 1. P_{td} is considered to determine the probability based on the delay (waiting) time on a proposal to see coalition formation, as T decreases—meaning less time required for the approval of a bill—the fraction increases, hence the probability of maintaining the same coalition also increase. T in this case is a proportional fraction between 0 and 1, where 0 is null time of waiting and 1 is a very long time of delay. Thus:

$$P(C_{t2} = C_{t1}|C_{t1}) = \frac{k \cdot (1 - I_{L_1} + I_{L_2})}{K} \quad \textbf{(5)}$$

$$P(C_{t2} = C_{t1}|C_{t1}) = \frac{e^{-\mu(T)} \cdot k}{K} \quad \textbf{(6)}$$

It is important to note that these probabilities are described to calculate a probabilistic value on C_{t2} for each of the exemplified TCs. Level-k reasoning plays a role in the incidence depending on the level of the legislators in the second period when returning the favor in the vote. It should be noted that the shapes of the functions may differ, and their values will depend on the legislative processes being attained at the time. It could depend on if we metric that in number of sessions for project approval or if it refers to days of delay. The level of reasoning will provide legislators with a modification in their preferences to vote together and achieve a logroll.

The k-levels are influential, on that account, in the utility gained by legislators in the first period and also in the probability that those benefiting from the first coalition decide to cooperate in a second period, even if it means disrupting the transitivity of preferences regarding legislative proposals: reaching a logrolling logic.

4. RESULTS

4.1. Interpretation and Implications

The probabilities derived above quantify the likelihood of coalition stability as a function of ideological proximity and procedural timing. In this framework, higher levels of cognitive reasoning and lower ideological distances or delays increase the chances of sustained cooperation. If the conditional probability of coalition re-formation exceeds a critical threshold—such as 0,5—the model predicts a stable logrolling arrangement across periods. Conversely, probabilities below that threshold suggest a breakdown in cooperation, with the coalition likely dissolving after the initial vote.

This outcome reflects the role of bounded rationality: legislators with limited depth of strategic reasoning may fail to anticipate higher-order threats or reciprocation strategies, leading to premature collapses in cooperation. It is important to emphasize, however, that this constitutes a theoretical approximation. The functional forms and probability distributions used here are illustrative and could vary significantly depending on the specific legislative body under consideration and its internal dynamics. As such, different institutional settings may yield distinct incentive structures, reasoning thresholds, or time-sensitivity parameters.

Nevertheless, this stylized model reinforces the central claim of the paper: that cognitive hierarchies and transaction costs meaningfully shape legislative cooperation. It also demonstrates how limited strategic reasoning may inhibit the formation of efficient coalitions, underscoring the value of integrating logrolling theory with behavioral and experimental approaches such as level-k reasoning.

5. CONCLUSIONS AND DISCUSSION

5.1. Discussion

The attainment scenario of logrolling is not simplified to a probabilistic approach determined by a specific voting strategy. Rather, it encompasses the evolving conceptions and preferences of legislators in a context where coordination within a coalition mutually benefits them. The assumption of a positive sum-game for logrolling formation is shaped by the voting design and the type of voting rule employed. A discursive argument is presented to justify the form of utility functions, probabilities of a repeated coalition and the strategic reasoning behind why coalitions should form to conceive logrolling.

The economic argument, when delving into distinctions in preferences and the incentives legislators have for decision-making, is extended far

beyond psychological determinants and can be attributed to several factors not encapsulated in a cognitive differentiation within legislators' minds. Consequently, additional elements need to be integrated with the level-k theory and its representation under TC to explore logrolling further.

To accelerate the argument that cognitive differences decisively impact stable and reciprocal voting strategies, it is essential to adequately define the environment where vote allocations converge to stable assignments, as described by Casella & Palfrey (2020) within strictly preferred coalitions when vote-trading exists. These allocations may be explained through attention biases, risk aversion, persuasive abilities, and more—all elements extensively studied in behavioral literature to estimate an economic effect not explained by classical utility maximization.

In a complex environment, axioms are indispensable for modeling the real world and deriving falsifiable interpretations. Wilson (1969) develops a model associating votes in social states exchange, aligning individual preferences regarding generalized legislative proposals. While experimentation could explore additional elements to construct indices and probability distributions guiding agents' decisions, studying vote-trading in the presence of TCs, specifically ideological distances and law processing delay time, could provide insights into the implications of different cognitive reasoning levels "*k*" in handling each case. That is why it is necessary to note the attribution of reasoning levels to previsualize possible gains for each agent.

As the argument for the first case was constructed, the fact that a legislator possesses a higher reasoning level allows them to perceive net benefits surpassing distributed costs (Buchanan & Tullock, 1965), serving as an impetus to vote in favor of an agreement. Clearly, the political context involves communication and some interests in legislative dynamics serve as reflected information or signaling for an explicit understanding of these benefits. This could be the case of a legislator abstaining from a project that doesn't favor them until a peer offers something more valuable. In this way, both would operate with a reasoning level guiding their decisions on whether to concede (despite differing ideologies) or form a coalition outside the legislative realm, focusing more on power distribution and favor return (Hortala-Vallve et. al, 2023).

An attention or persuasion bias could similarly influence vote structuring if a legislator with a higher level has the ability to influence the votes of lower-level legislators. Higher levels of reasoning might be associated with more sophisticated cognitive qualities enabling them to induce others into a situation more favorable to this player. Fortunately, in this modeling, what is more favorable for one player is also beneficial for the rest. However, in the context of temporal delay for the passage or rejection of a bill, the type of voting and the number of legislators in a congress become pivotal.

McGann (2019) posits that an institutional aspect could be the communication lines among legislators and the how discipline in the political party

is key to perceive the coalitions they can conform. Associating these qualities with varying levels-k of rationality could potentially lend credence to the hypothesis that a higher *k* might prompt a legislator to express their vote in favor of a coalition and dismantle the threshold incentive contract, described by Caplan (2006) as a minimal political benefit.

The number of legislators and the type of legislative body are crucial in discerning whether it's a "grand-logroll" situation or even an implicit logrolling scenario, where communication among legislators is almost absent. The directions these prescriptions would influence a legislator's decision depend on how much their *k* level determines their willingness to concede when the processing of a law is delayed in congress. The institutional cultivation preceding this logrolling, as expressed by François (2003) in their writings on political entrepreneurship, constitutes the existence of stable institutions and communicative processes of preferences. Although, high k levels already comprehend this to a greater extent, encouraging them to advocate for cooperation.

Establishing relations between cognitive levels of reasoning and the social characteristics possessed by legislators opens an empirical research avenue to determine whether these elements are significant enough to guide a logrolling situation. The conditionality between two consecutive periods with consequent responses must be a reality for this to occur, hence, cooperative games can be proposed where the probability of a coalition forming is subject to the influence of psychosocial qualities, political philosophical conceptions, and ultimately, the main legislative survival.

This paper considers elements such as attention and persuasion biases, institutional design, and the number of legislators to assess if a logrolling situation can be explained by strategic cognitive differences. However, the limitations of these theoretical intuitions and their mathematical modeling approaches are extensive and have been a subject of study in political science for decades. The configurations of legislative modus operandi can encompass myriad facets to explain strategic voting, thus the challenge lies in how well the level-k theory predicts vote exchanges.

5.2. Further Developments and Limitations

Within the realm of conceptual analysis and approaches to theoretical level-k reasoning, it is imperative to acknowledge the explanatory limitations inherent in this concept of legislative vote-trading. It is recognizable and evident that numerous factors beyond a cognitive difference among legislators account for the occurrence of logrolling situations. In political science literature, it is argued that market dynamics do not singularly shape the voting process and its outcomes. There exists a distinction between how a firm maximizes its profits and how legislative institutions operate, as explained by Weingast & Marshall (1988).

Furthermore, the conceptual application concerning transaction costs constitutes a very specific sphere for inferring reasoning levels, as hypothetical scenarios involving exchanges of favors between parties or representative blocs highlight the presence of extra-legislative transactions. These could take the form of favors being bought and sold between political parties, as proposed by Koford (1982), focusing on personal qualities of legislators such as leadership within their blocs. Additionally, situations may arise where legislators are compelled to act in specific ways due to the socio-political environment of a particular country, especially in semi-presidential regimes to bicameralism influences executive and judicial decisions.

The interest groups represented by legislators also warrant consideration when explaining why a voting outcome unfolds in a particular manner. Power distribution issues and inherent incentives to advocate for self-interests present the primary counterarguments against employing cognitive differentiation reasoning to solidify cooperative strategies. Potential infinite cycles in majority voting schemes could also lead to an equalization of reasoning levels, but verifying this hypothesis would necessitate laboratory experiments.

Another clarification, concerning transaction cost situations, is that delays in the processing of a law could be an endogenous phenomenon within each voting scheme. In such cases, it wouldn't serve as an explanation to resolve suboptimal equilibriums through reasoning but rather as an additional parameter within the same explanation of logrolling. Various functional forms could be attributed to reasons shaping a vote; however, only one of them is explored and linked within the literature—the legislator considering actions of others at a lower level. Exploring new functions that align with the schematization and assumptions of logrolling, which might not always fall within a positive sum-game, would thus be necessary.

These considerations form part of the constraints encountered when attempting to associate reasoning levels and logrolling more directly, especially when there are ideological distances and delays in proposing laws. It would be essential in the future to engage in this type of analysis to empirically confirm or, conversely, refute whether this combination of concepts across various economic branches is suitable for explaining legislative decisions.

The voting strategies can be driven by legislators' desire to reach agreements. Previously, this was understood as a situation of logrolling, where vote-trading can be explained by the natural and cognitive incentives of voters, providing formal research opportunities for theoretical and empirical proposals regarding the formation of these coalitions. In this context, an approach stemming from level-k reasoning has allowed us to theorize on how different levels of reasoning influence voting decisions.

The described scenario plays a role in incidence depending on the legislators' level in the second period when returning the favor in voting because it will provide legislators with a modification in their preferences to vote together and achieve a logroll. It's worth mentioning that assumptions arguing that a

fixed level of reasoning influences fostering cooperation when legislators have different ideologies or perceive time passing as a detrimental factor in cooperation are not intuitive and may connect with psychosocial qualities of legislators, such as persuasion capacity and the ability to lead legislative blocs.

BIBLIOGRAPHY

Arrow, K. J. (1983). Collected papers of Kenneth J. Arrow: Social choice and justice (Vol. 1). *Harvard University Press*.

Austen-Smith, D., y Banks, J. (1988). Elections, Coalitions, and Legislative Outcomes. *American Political Science Review, 82*(2), 405–422.

Bassi, A. (2015). Voting Systems and Strategic Manipulation: An Experimental Study. *Journal of Theoretical Politics, 27*(1), 58–85.

Bassi, A., y Williams, K. C. (2014). Examining Monotonicity and Saliency Using Level-k Reasoning in a Voting Game. *Games and Economic Behavior, 5*(1), 26–52.

Bentley, A. F. H. (1967). *The Process of Government*. Harvard University Press.

Bernholz, P. (1973). *Logrolling, arrow paradox and cyclical majorities*. Public Choice, 87-95.

Bernholz, P. (2012). From The Calculus of Consent to Extended Logrolling, Negative Externalities, and the Coase Theorem. *Public Choice, 152*(3–4), 265–271.

Buchanan, J. M., & Tollison, R. D. (Eds.). (1984). *The Theory of Public Choice II*. University of Michigan Press.

Buchanan, J. M., & Tullock, G. (1965). *The Calculus of Consent: Logical Foundations of Constitutional Democracy*. University of Michigan Press.

Caplan, B. (2006). *Designing Democracy: Ideas for Better Rules*.

Carrubba, C. J., y Volden, C. (2000). Coalitional Politics and Logrolling in Legislative Institutions. *American Journal of Political Science*, 261–277.

Casella, A., y Palfrey, T. R. (2021). Trading Votes for Votes: A Laboratory Study. *Games and Economic Behavior, 125*, 1–26.

Chappell Jr, H. W. (1981). Conflict of Interest and Congressional Voting: A Note. *Public Choice, 37*(2), 331–335.

Coleman, J. S. (1966). The Possibility of a Social Welfare Function. *The American Economic Review, 56*(5), 1105–1122.

De Clippel, G., Saran, R., y Serrano, R. (2019). Level-Mechanism Design. *The Review of Economic Studies, 86*(3), 1207–1227.

Enelow, J. M. (1986). The Stability of Logrolling: An Expectations Approach. *Public Choice, 51*(3), 285–294.

François, A. (2003). The Political Entrepreneur and the Coordination of the Political Process: A Market Process Perspective of the Political Market. *The Review of Austrian Economics, 16*(2–3), 153–168.

Gibbard, A. (1973). Manipulation of Voting Schemes: A General Result. *Econometrica: Journal of the Econometric Society*, 587–601.

Hortala-Vallve, R., Meriläinen, J., y Tukiainen, J. (2023). *Pre-Electoral Coalitions and the Distribution of Political Power*.

Ingham, S. (2016). Social Choice and Popular Control. *Journal of Theoretical Politics, 28*(2), 331–349.

Kau, J. B., y Rubin, P. H. (1979). Self-Interest, Ideology, and Logrolling in Congressional Voting. *The Journal of Law and Economics, 22*(2), 365–384.

Koford, K. J. (1982). Centralized Vote-Trading. *Public Choice, 39*(2), 245–268.

Laband, D. N. (1988). Transactions Costs and Production in a Legislative Setting. *Public Choice*, 183–186.

McKelvey, R. D. (1979). General conditions for global intransitivities in formal voting models. *Econometrica: journal of the Econometric Society*, 1085-1112.

McGann, A. J. (2019). Logrolling and Coalitions. En *The Oxford Handbook of Public Choice, 1*, 452.

Miller, N. R. (1977). Logrolling, Vote Trading, and the Paradox of Voting: A Game-Theoretical Overview. *Public Choice*, 51–75.

Nagel, R. (1995). Unraveling in Guessing Games: An Experimental Study. *The American Economic Review, 85*(5), 1313–1326.

Oppenheimer, J. (1979). Outcomes of Logrolling in the Bargaining Set and Democratic Theory: Some Conjectures. *Public Choice*, 419–434.

Qu, Y., y Cheung, S. O. (2012). Logrolling "Win–Win" Settlement in Construction Dispute Mediation. *Automation in Construction, 24*, 61–71.

Riker, W. H., y Brams, S. J. (1973). The Paradox of Vote Trading. *American Political Science Review, 67*(4), 1235–1247.

Stratmann, T. (1992). The Effects of Logrolling on Congressional Voting. *The American Economic Review*, 1162–1176.

Tullock, G. (1970). A Simple Algebraic Logrolling Model. *The American Economic Review, 60*(3), 419–426.

Tullock, G., y Brennan, G. (1981). Why So Much Stability. *Public Choice*, 189–204.

Weingast, B. R., y Marshall, W. J. (1988). The Industrial Organization of Congress; Or, Why Legislatures, Like Firms, Are Not Organized as Markets. *Journal of Political Economy, 96*(1), 132–163.

Wilson, R. (1969). An Axiomatic Model of Logrolling. *The American Economic Review, 59*(3), 331–341.

Wood, B. D., & Bohte, J. (2004). Political transaction costs and the politics of administrative design. The Journal of Politics, 66(1), 176-202.

Zhang, J. (2021). Level-k Reasoning in School Choice. *Games and Economic Behavior, 128*, 1–17.